Exercise 74

On May 7, 1992, the space shuttle *Endeavour* was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters.

(a) Use a graphing calculator or computer to find the cubic polynomial that best models the velocity of the shuttle for the time interval $t \in [0, 125]$. Then graph this polynomial.

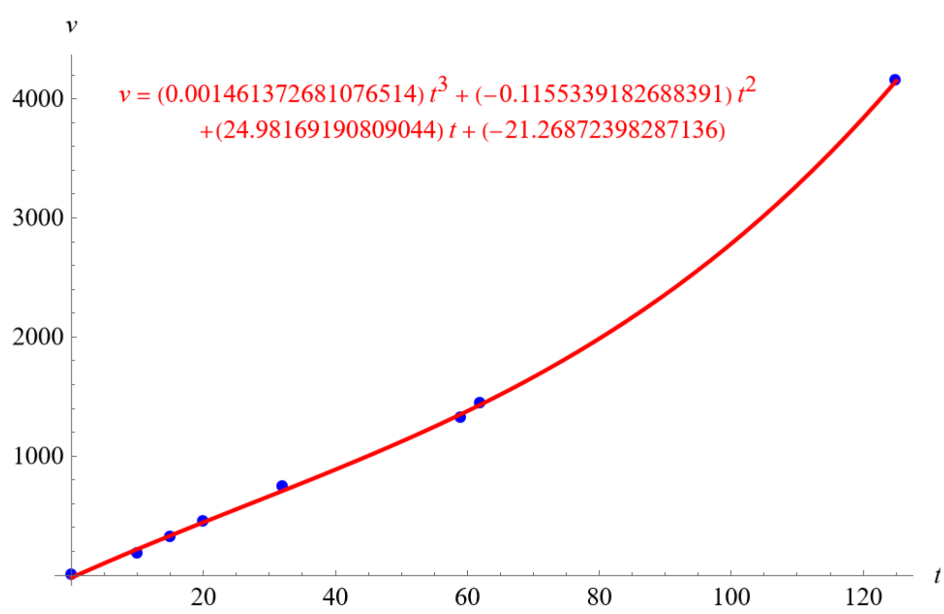
(b) Find a model for the acceleration of the shuttle and use it to estimate the maximum and minimum values of the acceleration during the first 125 seconds.

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

Solution

Part (a)

Plot the given data and use Mathematica's FindFit function to obtain the cubic function that best fits the data.



Part (b)

The model for acceleration is obtained by taking the derivative of the velocity.

$$\begin{aligned} a(t) &= v'(t) \\ &= \frac{d}{dt}(0.001461372681076514t^3 - 0.1155339182688391t^2 + 24.98169190809044t - 21.26872398287136) \\ &= 0.001461372681076514(3t^2) - 0.1155339182688391(2t) + 24.98169190809044(1) - 21.26872398287136(0) \\ &= 0.00438412t^2 - 0.231068t + 24.9817 \end{aligned}$$

To find the extreme values of $a(t)$ on $0 \leq t \leq 125$, take the derivative of $a(t)$.

$$\begin{aligned} a'(t) &= \frac{d}{dt}(0.00438412t^2 - 0.231068t + 24.9817) \\ &= 0.00438412(2t) - 0.231068(1) + 24.9817(0) \\ &= 0.00876824t - 0.231068 \end{aligned}$$

Then set $a'(t) = 0$ and solve for t.

$$0.00876824t - 0.231068 = 0$$

$$t = \frac{0.231068}{0.00876824} \approx 26.3528 \text{ seconds}$$

$t = 26.3528$ is within the interval $0 \leq t \leq 125$, so evaluate the function here.

$$a(26.3528) = 0.00438412(26.3528)^2 - 0.231068(26.3528) + 24.9817 \approx 21.9371 \ \frac{\text{ft}}{\text{s}^2} \quad \text{(absolute minimum)}$$

Evaluate the function at the endpoints.

$$a(0) = 0.00438412(0)^2 - 0.231068(0) + 24.9817 = 24.9817 \ \frac{\text{ft}}{\text{s}^2}$$

$$a(125) = 0.00438412(125)^2 - 0.231068(125) + 24.9817 = 64.6001 \ \frac{\text{ft}}{\text{s}^2} \quad \text{(absolute maximum)}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $0 \leq t \leq 125$.

The graph of the acceleration function below illustrates these results.